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Proton Mass Shift in Muonic Hydrogen Atom

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We show that the value of the proton mass depends on each bound state of muonic or electronic hydrogen atom. The charged particle bound to the proton produces magnetic field inside the proton. This makes a change to the amount of chiral condensate inside the proton. The change gives rise to the shift in the value of the proton mass. Numerically, the shift in the $2S$ state of the muonic hydrogen atom can be of the order of 0.1 MeV. The effect may solve the puzzle of the proton radius.

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I. INTRODUCTION

The proton mass would be much smaller than 1 GeV if the chiral symmetry was not spontaneously broken in QCD. It would take a value of several MeV. However the real mass is about 1 GeV which results from the spontaneous breaking of the chiral symmetry. The breaking generates the chiral condensate $\langle \bar{q}q \rangle \simeq (-240\text{MeV})^3$, which leads to the large proton mass. It has been discussed[1, 2] that the chiral condensate grows with external magnetic field. Since the proton has the magnetic moment, it has an intrinsic magnetic field inside the proton itself. Thus, the amount of the chiral condensate inside the proton is larger than that of the condensate in vacuum. Furthermore, an electron or a muon bound to the proton produces a magnetic field inside the proton, which makes a change to the amount of the chiral condensate inside the proton. Although the change is much small, it causes the shift in the value of the proton mass.

In this paper we show that the magnetic field produced by a muon or an electron bound to the proton gives rise to the measurable shift in the value of the proton mass. The shift in the muonic hydrogen atom is much larger than that in the electronic hydrogen atom. This is because the Bohr radius in the muonic hydrogen atom is much smaller than the Bohr radius in the electronic hydrogen atom. Furthermore, the amount of the shift depends on each bound state. As the charged particle is closer to the proton, it produces stronger magnetic field inside the proton. Thus, the shift is larger in S states than that in the other states with higher orbital angular momentum $l \geq 1$. We point out that the recent puzzle[3, 4] of the proton radius may be solved by taking account of the proton mass shift in the muonic hydrogen atom.

The condensate inside the proton is exposed to two types of magnetic fields. The one is intrinsic magnetic field B_i and the other one is external magnetic field δB . The intrinsic magnetic field is its own magnetic field inside the proton because the proton has a magnetic moment. The external magnetic field is produced by an electron or muon bound to the proton. The strength of the external magnetic field depends on each bound state of the particles. As naively expected, the external magnetic field is much smaller than the intrinsic one. These magnetic fields change the amount of the chiral condensate so that they change the value of the proton mass. Although the proton mass shift caused by the external magnetic field is very small, we can measure the proton mass shift by observing emission spectrum of muonic or electronic hydrogen atom.

In order to find the mass shift caused by the magnetic field of the bound muon or electron, we first represent the proton mass M_P in term of the chiral condensate. We assume that the Ioffe's formula[5] holds under background magnetic field $\vec{B} = \vec{B}_i + \delta \vec{B}$

$$M_P = \frac{-4\pi^2 \langle \bar{q}q \rangle_B}{\Lambda_{QCD}^2} \quad (1)$$

where Λ_{QCD} denotes a constant characterizing QCD scale; $\Lambda_{QCD} \simeq 1\text{GeV}$. We use the units of $c = 1$ and $\hbar = h/2\pi = 1$. The formula is not precisely but approximately derived by using QCD sum rule. The question how the proton mass depends on the condensate is still controversial. Hence, our estimation of the proton mass shift with the use of the formula in eq(1) should be taken as being not seriously quantitative, only approximately quantitative. The point in the formula is that the proton mass depends on the magnetic field only through the chiral condensate. Thus, the effect of the small external magnetic field δB added to B_i on the proton mass is given such that

$$\delta M_{\delta B} = M_P \frac{\delta \langle \bar{q}q \rangle_{B_i}}{\langle \bar{q}q \rangle_{B_i}}, \quad \delta \langle \bar{q}q \rangle_{B_i} \equiv \langle \bar{q}q \rangle_{B_i + \delta B} - \langle \bar{q}q \rangle_{B_i}. \quad (2)$$

We note that in the case that $M_P \propto (-\langle \bar{q}q \rangle_B)^{1/3}$ as anticipated from dimensional analysis, the above formula is rewritten as $\delta M_{\delta B} = M_P \delta \langle \bar{q}q \rangle_{B_i} / 3 \langle \bar{q}q \rangle_{B_i}$. The difference between these two formulas is the factor 1/3. Hence, owing to the ambiguity of the dependence of the proton mass on the chiral condensate, subsequent our result of the proton mass shift has an error of the order of a numerical factor.

In addition to the Ioffe's formula, we assume that the intrinsic magnetic field \vec{B}_i inside the proton is proportional to the proton magnetic moment $\vec{\mu}_p = g_p \vec{S}_p e / 2M_P$, i.e. $\vec{B}_i \propto \vec{\mu}_p$, where \vec{S}_p denotes the proton spin and $g_p \simeq 5.6$. In principle, we can precisely determine the value B_i in the lattice gauge theory, although it has not yet performed. Instead, we may obtain the approximate value B_i such that $B_i \simeq g_p e / 8\pi r_p^3 M_P \sim (29\text{MeV})^2$ with the proton radius $r_p = 0.88\text{fm}$, assuming that a circular current I with its radius r_p has the magnetic moment $\mu_p = \pi r_p^2 I$ ($= g_p e / 4M_P$) and generates a magnetic field $B_i = I / 2r_p$ at the center of the circle. But, in this paper we take it as one of unknown parameters. The magnetic field $\delta \vec{B}$ produced by the electron or muon bound to the proton is added to this intrinsic magnetic field.

Secondly in order to calculate the mass shift in eq(2), we need an explicit formula representing the dependence of the chiral condensate on the background magnetic field. It has been shown[2] that the condensate depends on B such that

$$\langle \bar{q}q \rangle_B = \langle \bar{q}q \rangle_{B=0} \left(1 + \frac{(eB)^2}{96\pi^2 F_\pi^2 m_\pi^2} \right), \quad (3)$$

in the limit $eB \ll m_\pi^2$, where $F_\pi \simeq 90\text{MeV}$ (m_π) denotes pion decay constant (pion mass). The formula has been explicitly derived by using chiral perturbation theory[6], i.e., low energy model of hadrons. It has been partially confirmed in lattice gauge theories[7] where the value of the magnetic field is limited to be in the range $(180\text{MeV})^2 < eB < (700\text{MeV})^2$ and the pion mass is given by $m_\pi \simeq 200\text{MeV}$. That is, the quadratic dependence on B has been confirmed although B is limited to the range. But, the coefficient of the term $\propto B^2$ has been found to be smaller in the lattice gauge theories compared with the theoretical prediction in eq(3). Thus, the formula in eq(3) is not still confirmed. Hence, we take the value $F_\pi^2 m_\pi^2$ or F_π^2 as one of unknown parameters, although we use the formula in eq(3) even in the case of the small magnetic field $B \sim (29\text{MeV})^2 \ll m_\pi^2$.

Using the formula, we obtain the mass shift caused by a bound muon or electron

$$\delta M_{\delta B} = M_P \frac{e \vec{B}_i \cdot e \delta \vec{B}}{48\pi^2 F_\pi^2 m_\pi^2} \quad (4)$$

The magnetic field $\delta \vec{B}$ produced by the bound muon or electron is given by

$$\delta \vec{B} = \mu_{B,\mu,e} \frac{\vec{l}_{\mu,e}}{4\pi r^3} + \frac{1}{4\pi r^3} \left(\frac{3(\vec{\mu}_{\mu,e} \cdot \vec{r})\vec{r}}{r^2} - \vec{\mu}_{\mu,e} \right) + \frac{2\vec{\mu}_{\mu,e}}{3} \delta^3(r), \quad (5)$$

where $\vec{\mu}_{\mu,e}$ ($= -g_{\mu,e} \mu_{B,\mu,e} \vec{S}_{\mu,e}$) denotes the magnetic moment of the muon or electron with $\mu_{B,\mu,e} \equiv e / 2m_{\mu,e}$ and $g_{\mu,e} \simeq 2$; $\vec{S}_{\mu,e}$ denotes the spin of the muon or electron. The first term represents a contribution of the orbital angular momentum \vec{l} , and the second and third terms represent contributions of the magnetic moment $\vec{\mu}_{\mu,e}$. The muon or electron is located at $\vec{r} = 0$ in the coordinate.

Therefore, taking the quantum average with the use of the wavefunction of the muon or electron, the proton mass shift in eq(4) is given by

$$\langle \delta M_{\delta B} \rangle = \frac{e M_P}{48\pi^2 F_\pi^2 m_\pi^2} \left(\left\langle \frac{\mu_{B,\mu,e} e \vec{B}_i \cdot \vec{l}_{\mu,e}}{4\pi r^3} \right\rangle + \left\langle \frac{1}{4\pi r^3} \left(\frac{3(\vec{\mu}_{\mu,e} \cdot \vec{r})(e \vec{B}_i \cdot \vec{r})}{r^2} - e \vec{B}_i \cdot \vec{\mu}_{\mu,e} \right) \right\rangle + \left\langle \frac{2e \vec{B}_i \cdot \vec{\mu}_{\mu,e}}{3} \delta^3(r) \right\rangle \right), \quad (6)$$

where the coordinate \vec{r} denotes the location of the bound particles; the proton is located at $\vec{r} = 0$.

Since the first and second terms vanish for the S states with $l = 0$, we can easily obtain the mass shift in the S states with the principal quantum number n ,

$$\langle \delta M_{\delta B} \rangle = \frac{eM_P}{48\pi^2 F_\pi^2 m_\pi^2} \frac{-2eB_i \mu_{\mu,e} |\Psi_n^{l=0}(\vec{r}=0)|^2}{3} \left(F(F+1) - \frac{3}{2} \right) \quad (7)$$

with $\mu_{\mu,e} \equiv g_{\mu,e}e/2m_{\mu,e}$ and total spin $F = 1$ or $F = 0$ of the proton and muon (electron), where we have set $\vec{B}_i \equiv 2B_i \vec{S}_p$. We have used nonrelativistic hydrogen wave functions $\Psi_n^{l=0}(\vec{r})$ with angular momentum $l = 0$, principal quantum number n and reduced mass $\bar{m}_{\mu,e} \equiv m_{\mu,e}M_P/(M_P + m_{\mu,e})$.

For example, in the state $2S_{1/2}^{F=1}$ ($n=2$) of the muonic hydrogen atom

$$\langle \delta M_{\delta B}(\text{muon}) \rangle_{2S} = \frac{eM_P}{48\pi^2 F_\pi^2 m_\pi^2} \frac{-eB_i \mu_\mu (\alpha \bar{m}_\mu)^3}{24\pi} = \frac{-\alpha^4 \bar{m}_\mu^3 M_P}{288\pi^2 m_\pi^2 m_\mu} \frac{eB_i}{F_\pi^2} \simeq -0.39 \frac{eB_i}{F_\pi^2} \text{ meV}, \quad (8)$$

with $\alpha \equiv e^2/4\pi \simeq 1/137$, where the values $M_P = 938\text{MeV}$, $m_\mu = 106\text{MeV}$ and $m_\pi = 140\text{MeV}$ are used. The factor eB_i/F_π^2 is an unknown parameter. The negative value $\langle \delta M_{\delta B} \rangle_{2S}$ comes from the fact that the magnetic moment of the proton is anti-parallel to the magnetic moment of the muon in the state $2S_{1/2}^{F=1}$. Similarly, we may estimate the mass shifts in the other S states by the explicit use of their wave functions. We should notice that as naively expected, the external magnetic field $e\delta B = \alpha^4 \bar{m}_\mu^3/6m_\mu \sim (10^{-3}\text{MeV})^2$ extracted from the above formula is much smaller than the intrinsic magnetic field $eB_i \sim (17\text{MeV})^2$.

Furthermore, in order to estimate the mass shifts in the states with higher angular momenta $l \geq 1$ we only need to estimate the first and second terms in eq(6) since the third term vanishes. For example, we shall estimate the proton mass shift in the state $2P_{3/2}^{F=2}$ of the muonic hydrogen atom where the \vec{F} denotes the total angular momentum of the proton and the muon; $\vec{F} = \vec{S}_p + \vec{S}_\mu + \vec{l}_\mu$. In the calculation we take the average of $\langle \vec{S}_p \cdot \vec{l}_\mu \rangle$ and $\langle \frac{3(\vec{S}_\mu \cdot \vec{r})(\vec{S}_p \cdot \vec{r})}{r^2} - \vec{S}_\mu \cdot \vec{S}_p \rangle$ over the spin and the angular coordinates

$$\langle \vec{S}_p \cdot \vec{l}_\mu \rangle = \frac{1}{2}, \quad \langle \frac{3(\vec{S}_\mu \cdot \vec{r})(\vec{S}_p \cdot \vec{r})}{r^2} - \vec{S}_\mu \cdot \vec{S}_p \rangle = -\frac{1}{10}. \quad (9)$$

Finally taking the average of $1/r^3$ over the radial coordinate r with the explicit use of the wave function $\Psi_{n=2}^{l=1}(\vec{r})$, we obtain

$$\langle \delta M_{\delta B}(\text{muon}) \rangle_{2P} = \frac{7\alpha^4 \bar{m}_\mu^3 M_P}{11520\pi^2 m_\pi^2 m_\mu} \frac{eB_i}{F_\pi^2} \simeq 0.068 \frac{eB_i}{F_\pi^2} \text{ meV}. \quad (10)$$

where the identical values M_P , m_μ and m_π to those in eq(8) was used. As we have expected, we find that the mass shift in the S state is much larger than that in the P state. In this way we can evaluate the proton mass shift in any bound states of the muon or electron by using the formula in eq(6). We should notice that the mass shift is caused by non-perturbative effect, since the nonvanishing chiral condensate arises owing to the spontaneous breaking of the chiral symmetry. Thus, the proton mass shift has not been previously found in any perturbative analyses of the muonic hydrogen spectra.

Now, we point out a possible solution for the puzzle[3, 4] of the proton radius. The puzzle arises from the discrepancy between the theoretical prediction of the photon energy and the observed one of the muonic transition in the muonic hydrogen atom,

$$\left(E(2P_{3/2}^{F=2}) - E(2S_{1/2}^{F=1}) \right)_{\text{obs}} - \left(E(2P_{3/2}^{F=2}) - E(2S_{1/2}^{F=1}) \right)_{\text{th}} = 0.31 \text{ meV} \quad (11)$$

where a muon emits a photon in the transition from the state $2P_{3/2}^{F=2}$ with the energy $E(2P_{3/2}^{F=2})$ to the state $2S_{1/2}^{F=1}$ with the lower energy $E(2S_{1/2}^{F=1})$. The CODATA-2010 value of the proton radius $r_p \simeq 0.88\text{fm}$ was used in the theoretical prediction, which has been obtained [8] from the analysis of the electronic hydrogen spectra and the experiment of electron-proton scattering. On the other hand, the theoretical prediction agrees with the observed one when one uses the smaller radius $r_p \simeq 0.84\text{fm}$. The theoretical prediction of the spectrum in the muonic hydrogen has been examined[4] in detail, but there have been no rooms found for the improvement except for the change of the proton radius. This is the puzzle of the proton radius.

Our proposal for the solution of the puzzle is that the discrepancy in eq(11) may be solved by taking account of the proton mass shift caused by the magnetic field of the bound muon. Namely, when the proton mass shift is taken into account, the above formula is improved such that

$$\left(E(2P_{3/2}^{F=2}) - E(2S_{1/2}^{F=1})\right)_{\text{obs}} - \left(E(2P_{3/2}^{F=2}) - E(2S_{1/2}^{F=1})\right)_{\text{th}} = 0.31 \text{ meV} - 0.46 \frac{eB_i}{F_\pi^2} \text{ meV}, \quad (12)$$

with the proton radius $r_p \simeq 0.88 \text{ fm}$ being used. In order to determine the parameter eB_i/F_π^2 , we need to obtain the intrinsic magnetic field B_i and the coefficient $F_\pi^2 m_\pi^2$ in eq(3), both of which have not yet obtained in lattice gauge theories. When we tentatively adopt the approximate value $B_i \sim (30 \text{ MeV})^2$ and use the pion decay constant $F_\pi \simeq 90 \text{ MeV}$, we obtain $eB_i/F_\pi^2 \sim 0.04$. Hence, the mass shift is slightly too small to solve the puzzle. But we should remember that there are several ambiguities of the physical parameters B_i , $F_\pi^2 m_\pi^2$ and of the dependence of the proton mass on the chiral condensate in eq(1). In addition to the ambiguities, we need to reexamine the analysis of the electronic hydrogen spectra to obtain the proton radius r_p by taking into account the proton mass shift. Then, we find that when we include the effect of the proton mass shift in the analysis of the electronic hydrogen spectra, we obtain the proton radius r_p smaller than the value 0.88 fm which was obtained without the inclusion of the effect of the mass shift. The smaller proton radius leads to the discrepancy with smaller amount than 0.31 meV in eq(11). Then, the small parameter $eB_i/F_\pi^2 \sim 0.04$ obtained approximately would be suitable for the discrepancy.

The proton mass shift is fairly small in the case of the electronic hydrogen atom because the Bohr radius of the electron is 200 times larger than that of the muon. Actually, the shift can be easily obtained by replacing the muon mass by the electron mass in eq(8) and eq(10),

$$\begin{aligned} \langle \delta M_{\delta B}(\text{electron}) \rangle_{2S} &= \frac{\alpha^4 \bar{m}_e^3 M}{288 \pi^2 m_\pi^2 m_e} \frac{eB_i}{F_\pi^2} \simeq -1.2 \times 10^{-5} \frac{eB_i}{F_\pi^2} \text{ meV} \\ \langle \delta M_{\delta B}(\text{electron}) \rangle_{2P} &= \frac{7 \alpha^4 \bar{m}_e^3 M}{11520 \pi^2 m_\pi^2 m_e} \frac{eB_i}{F_\pi^2} \simeq 2.2 \times 10^{-6} \frac{eB_i}{F_\pi^2} \text{ meV} \end{aligned} \quad (13)$$

where the value $m_e = 0.51 \text{ MeV}$ is used. These values are very small, but they are measurable because the accuracy of the observed spectrum is smaller than the values.

To summarize, we have shown that the value of the proton mass in atoms is different from that in vacuum. This is because the chiral condensate inside the proton is influenced by the magnetic field of bound electrons or muons. We have calculated the shifts in the value of the proton mass in the muonic or electronic hydrogen atom. In particular we have explicitly shown the mass shifts in the bound states $2S_{1/2}^{F=1}$ and $2P_{3/2}^{F=2}$ of the muonic and electronic hydrogen atom. We have found that the mass shifts give rise to a possible solution for the puzzle of the proton radius.

Taking account of the proton mass shift, the values of the fine structure constant $\alpha = e^2/4\pi$, Rydberg constant $R_\infty \equiv \alpha^2 m_e/4\pi$ and the proton radius r_p have to be reexamined in the analysis of the electronic hydrogen spectra.

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